DewTwo: a transparent PCS with small proofs from falsifiable assumptions

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Polynomial Commitment Schemes

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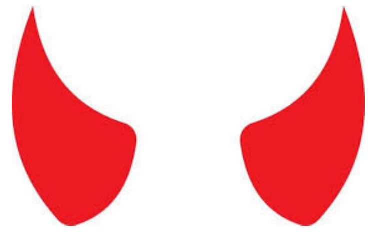

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Transparent PC schemes

Scheme	Category	Prover time	Verifier time	Proof Size	Falsifiable?	Transparent?
WHIR [ACFY25]	Hash-Based	$O(N \log(N))$ $\mathbb{F}$	$O(\lambda \log(N) \log\log(N))$ $\mathbb{F}$	$\lambda \log(N) \log\log(N)$ $\mathbb{F}$ 107 KB	Yes	Yes
Bulletproofs [BBBPWM18]	DLOG-Based	$O(N)$ $\mathbb{G}_{EC}$	$O(N)$ $\mathbb{G}_{EC}$	$2 \log(N)$ $\mathbb{G}_{EC}$ 1.5 KB	Yes	Yes
Dory [Lee21]	DLOG-Based	$O(N)$ $\mathbb{G}$	$O(\log N)$ $\mathbb{G}$	$6 \log(N)$ $\mathbb{G}_T$ 37 KB	Yes	Yes
Dew, Behemoth [AGLMS23, SB23]	Groups of unknown order	$O(N)$ $\mathbb{G}_{GUO}$ $O(N^3 \log N)$ $\mathbb{B}$	$O(\log N)$ $\mathbb{F}$ $O(1)$ $\mathbb{G}_{GUO}$	$O(1)$ $\mathbb{G}_{GUO}$ 9~12 KB	No	Yes

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DewTwo **[This work]**	Groups of unknown order	$O(N)\ \mathbb{G}_{GUO}$ $O(N \log^2 N)\ \mathbb{B}$	$O(\log N)\ \mathbb{F}$ $O(\log N)\ \mathbb{G}_{GUO}$	$\log\log(N)\ \mathbb{G}_{GUO}$ 4.5 KB	Yes	Yes

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- **Our work:** New falsifiable assumption in GUO instead of Generic Group Model.

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$$= 324 \in \mathbb{Z}$$

Open and Verify

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$$:= s + \left(\tilde{p}_0\tilde{r}_{N-1} + \tilde{p}_1\tilde{r}_{N-2} + \ldots + \tilde{p}_{N-1}\tilde{r}_0\right)\cdot\alpha^{N-1} + u\cdot\alpha^N$$

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Strawman Construction

Open(ck, cm, p, z) :

Recall that cm $= \tilde{p}(\alpha) \cdot G$

Verify(vk, cm, z, y, π) :

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s, t, u are too large to send in the clear!

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$$\Updownarrow$$

$$(t - a)(b - t) \geq 0$$

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Proof size $O(\log \log N)$

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Conclusion

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  - Would imply very efficient post-quantum PCS.

Thank you for listening!

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